Int. J. Heat Mass Transfer. Vol. 22, pp. 1533–1538.

0017-9310/79/1101-1533 $02.00/0

THE RESPONSE OF A WELD POOL TO PERTURBATIONS IN POWER

J. G. ANDREWS and D. R. ATTHEY
Central Electricity Generating Board, Marchwood Engineering Laboratories,
Marchwood, Southampton, Hampshire, England

(*Received* 15 *November* 1978 *and in revised form* 15 *March* 1979)

Abstract—Automatic control of welding processes is rapidly developing and it is important to understand how the boundary of the weld pool responds to changes in the power. This paper shows how the response time depends on latent heat and the traverse speed of the heat source. For a slowly-traversing source, the solution is shown to be a superposition of the transient solution for a stationary source and the steady solution for a moving source. When the Peclet number, $\alpha = va/2D$ (v = traverse speed, a = equilibrium radius of a stationary pool, D = thermal diffusivity), is of $O(1)$ the amplitude and response time vary considerably from the front to the rear of the pool.

NOMENCLATURE

T, temperature;
T_m, melting point;
r, radial coordinate;
a, equilibrium radius of pool;
t, time;
$s(t)$, radius of weld pool at time t;
D, diffusion coefficient;
L, latent heat;
c, specific heat;
k, thermal conductivity;
ρ, density;
x, coordinate measured in direction of motion of heat source;
θ, angular coordinate measured from direction of motion;
n, outward normal to pool boundary;
v, speed of material relative to heat source;
p, Laplace transform parameter;
q, strength of heat source.

Non-dimensional parameters

α, Peclet number ($va/2D$);
u, $= T/T_m$;
R, $= r/a$;
S, $= s/a$;
τ, $= Dt/a^2$;
γ, $= L/cT_m$;
ε, fractional jump in power;
X, $= x/a$;
w, $= ue^{\alpha x}$;
$Q_1(\tau)$, power function.

1. INTRODUCTION

THE RAPID progress in welding technology has stimulated considerable scientific interest in the processes involved. In practice, the weld pool boundary appears to be sensitive to small variations in various parameters, e.g. power, traverse speed, etc., and knowledge of its shape is important to ensure satisfactory welds. In this paper we are concerned with the effect of a change in power on the weld pool boundary and we include the effect of latent heat for problems which are analytically tractable.

We begin by considering the response due to a stationary source of heat and then treat the extra complication due to motion of the source. Analytic solutions for these cases can be found provided that both the fractional change in power and the Peclet number, $\alpha = va/2D$, are small, where v is the traverse speed of the source, a is the equilibrium radius of a stationary pool and D is the thermal diffusivity of the material. We also consider the problem where the Peclet number is not necessarily small, with latent heat neglected, as this case can also arise in some welding conditions. We also assume (a) a point source of heat; (b) uniform thermophysical properties everywhere; (c) semi-infinite geometry; and (d) no fluid motion in the pool.

Our Peclet number, α, is identical with Christiansen's "n" parameter [1] and is a measure of the distortion of the weld pool from a hemisphere due to motion of the source.

2. RESPONSE FOR A STATIONARY SOURCE WITH LATENT HEAT INCLUDED

In this section we consider a stationary source of heat. Initially the strength of the source q is constant, and at time $t = 0$ it is changed to $(1+\varepsilon)q$. The problem is to calculate the transient temperature variation and the response of the weld pool boundary, $r = s(t)$. For convenience, we normalise the variables as

$$u = T/T_m, \quad R = r/a, \quad S = s/a,$$
$$\tau = Dt/a^2, \quad \gamma = L/cT_m,$$

where T_m is the melting point, a is the equilibrium radius of the pool given by

$$a = q/2\pi k T_m,$$

k is the thermal conductivity, $D = k/\rho c$ is the thermal diffusivity, ρ and c are the density and specific heat of the material, respectively, and L is the latent heat of fusion. The ambient temperature is taken to be zero, for simplicity. The equation for unsteady heat flow is

$$(1/R^2)\partial(R^2\partial u/\partial R)/\partial R = \partial u/\partial\tau, \tag{1}$$

and initially the temperature distribution is given by

$$u_0 = 1/R. \tag{2}$$

For $\tau > 0$ we must solve equation (1) subject to the boundary conditions

$$\lim_{R\to 0}\{-R^2\partial u/\partial R\} = 1+\varepsilon, \tag{3}$$

$$\left.\begin{array}{l} u = 1 \quad (4) \\ [\partial u/\partial R]_L^S = \gamma\, dS/d\tau \quad (5) \end{array}\right\} \text{ on } R = S(\tau),$$

and

$$u \to 0 \quad \text{as} \quad R \to \infty. \tag{6}$$

$[\ ]_L^S$ denotes the jump in heat flux across the melting boundary between the liquid and the solid. An exact solution of this problem does not seem possible but if we restrict our attention to small changes in power, i.e. $|\varepsilon| \ll 1$, then we can try a perturbation expansion. Setting

$$u(R,\tau) = 1+\varepsilon u_1(R,\tau), \tag{7}$$

$$S(\tau) = 1+\varepsilon S_1(\tau), \tag{8}$$

equations (1) and (3)–(6) become, to first order in ε,

$$(1/R^2)\partial(R^2\partial u_1/\partial R)/\partial R = \partial u_1/\partial\tau, \tag{9}$$

$$\lim_{R\to 0}(-R^2\partial u_1/\partial R) = 1,$$

$$\left.\begin{array}{l} u_1 = S_1 \quad (10) \\ [\partial u_1/\partial R] = \gamma\, dS_1/d\tau \quad (11) \end{array}\right\} \text{ on } R = 1,$$

$$u_1 \to 0 \quad \text{as} \quad R\to\infty,$$

together with the initial condition $u_1(R,0) = 0$. Since $|\varepsilon| \ll 1$, i.e. the boundary is always close to its equilibrium position, we have been able to apply Taylor's theorem about $R = 1$, thus replacing the conditions on the *moving* boundary, (4) and (5), by (10) and (11) on the *fixed* boundary $R = 1$. This simplification makes the problem linear and amenable to Laplace transforms. We obtain the following conditions on the transformed variables:

$$(1/R^2)\,d(R^2\,d\bar{u}_1/dR)/dR = p\bar{u}_1, \tag{12}$$

$$\lim_{R\to 0}(-R^2\,d\bar{u}_1/dR) = 1/p, \tag{13}$$

$$\left.\begin{array}{l} \bar{u}_1 = \bar{S}_1 \quad (14) \\ [d\bar{u}_1/dR] = \gamma p\bar{S}_1 \quad (15) \end{array}\right\} R = 1,$$

$$\bar{u}_1 \to 0 \quad \text{as} \quad R\to\infty, \tag{16}$$

where the bar denotes the Laplace transform. The general solution of (12) is

$$\bar{u}_1 = (A\,e^{Rp^{1/2}} + B\,e^{-Rp^{1/2}})/R.$$

Choosing solutions of the above form in both the liquid and solid regions, applying the boundary conditions (13)–(16), we find

$$\bar{S}_1 = e^{-p^{1/2}}/\{p+\tfrac{1}{2}\gamma p^{3/2}(1-e^{-2p^{1/2}})\}. \tag{17}$$

This transform does not appear in standard tables but we can look at its asymptotic behaviour. For large p (i.e. small times), (17) approximates to

$$\bar{S}_1 = e^{-p^{1/2}}/(p+\tfrac{1}{2}\gamma p^{3/2}),$$

which can be inverted to give

$$S_1(\tau) = \operatorname{erfc}(\tfrac{1}{2}\tau^{-1/2}) - e^{2(1+2\tau/\gamma)/\gamma}\operatorname{erfc}\{(\tfrac{1}{2}\tau^{-1/2})+2\tau^{1/2}/\gamma\}. \tag{18}$$

For small p (i.e. large times), the binomial theorem gives

$$\bar{S}_1 = p^{-1}e^{-p^{1/2}}\{1-\tfrac{1}{2}\gamma p^{1/2}(1-e^{-2p^{1/2}}) + \tfrac{1}{4}\gamma^2 p(1-e^{-2p^{1/2}})^2 - \dots\}.$$

Each term in the series can be inverted using standard tables, yielding

$$S_1(\tau) = \phi_0(\tau)+\gamma\phi_1(\tau)+\gamma^2\phi_2(\tau)+\dots, \tag{19}$$

where

$$\phi_0(\tau) = \operatorname{erfc}(\tfrac{1}{2}\tau^{-1/2}),$$
$$\phi_1(\tau) = -\tfrac{1}{2}e^{-1/4\tau}(1-e^{-2/\tau})/(\pi\tau)^{1/2},$$
$$\phi_2(\tau) = e^{-1/4\tau}(1-6\,e^{-2/\tau}+5\,e^{-6/\tau})/8(\pi\tau^3)^{1/2}.$$

Figures 1 and 2 show the variation of $S_1(\tau)$ with τ for $\gamma = 0$, 0.3, 0.5, 1.0 over the ranges $0 \leqslant \tau \leqslant 0.8$ and $0 \leqslant \tau \leqslant 6$, respectively. The match between the small and large time solutions around $\tau = 1$ is extremely good.

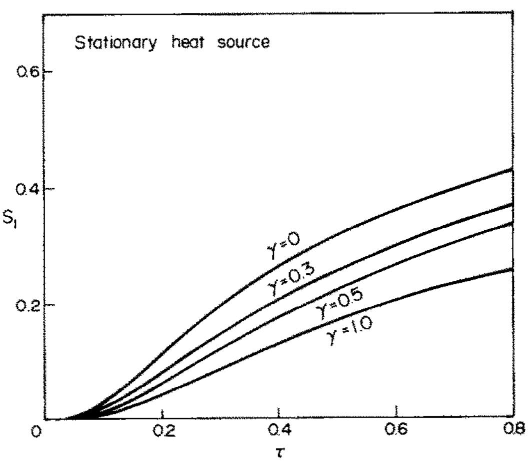

FIG. 1. Variation of normalized radius of molten boundary with normalized time, for small times.

3. RESPONSE FOR A SLOWLY-MOVING SOURCE WITH LATENT HEAT INCLUDED

We now consider the added complication of the motion of the source. For simplicity, we take the coordinate frame to be at rest relative to the source, with the material streaming past with uniform speed, v, in the negative x direction. The heat conduction

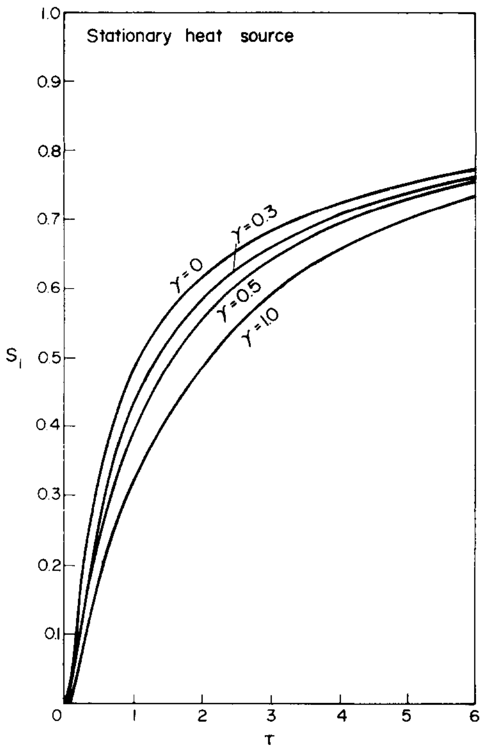

FIG. 2. Variation of normalized radius of molten boundary with normalized time, for large times.

equation in these moving co-ordinates is

$$D\nabla^2 T = \partial T/\partial t - v\partial T/\partial x, \tag{20}$$

and the discontinuity condition on the melting boundary is

$$[k\partial T/\partial n] = L\rho\{v\cos\theta + (\partial s/\partial t)\hat{\mathbf{r}}\cdot\hat{\mathbf{n}}\}, \tag{21}$$

on $r = s(\theta, t)$, where $\mathbf{n}$ is the outward normal to the boundary. The remaining boundary conditions are equivalent to those for the stationary case.

The general problem of a moving source, with latent heat included, is very difficult. Even the steady-state case requires a numerical solution [2] and the unsteady case is still more complicated [3]. However, when the Peclet number, $\alpha = va/2D$, is sufficiently small, it is possible to use a perturbation approach to determine the steady-state solution [4]. If we make the further assumption that the fractional change in power is small, i.e. $|\varepsilon| \ll 1$, then we may solve for the transient response with latent heat included. We shall assume that both α and ε are of the same order and neglect second order terms.

Following Malmuth [4] we define a dimensionless temperature function

$$w = (T/T_m)\,e^{\alpha X}, \tag{22}$$

and we are required to solve

$$\nabla'^2 w - \alpha^2 w = \partial w/\partial\tau, \tag{23}$$

where ∇'^2 is the dimensionless Laplace operator, and subject to the boundary conditions

$$\lim_{R\to 0}\,(-R^2\partial w/\partial R) = 1+\varepsilon, \tag{24}$$

$$\left.\begin{aligned} &w = e^{\alpha X} \quad &(25)\\ &[\partial w/\partial R]_L^S = \gamma\, e^{\alpha X}(2\alpha\cos\theta + \partial S/\partial\tau) \quad &(26)\end{aligned}\right\}\text{ on } R = S(\tau),$$

$$\lim_{R\to 0} w = o(e^{\alpha X}), \tag{27}$$

and the initial condition given by the steady-state solution with the normalised power equal to unity. In obtaining equation (26) we have made the simplifying assumption that the pool boundary is only slightly distorted from a hemisphere.

To first order, we expand

$$w = w_0 + \varepsilon w_\varepsilon + \alpha w_\alpha, \tag{28}$$

and

$$S = S_0 + \varepsilon S_\varepsilon + \alpha S_\alpha. \tag{29}$$

Clearly the zero order solution is

$$w_0 = 1/R,$$

and

$$S_0 = 1.$$

The problem for $w_\varepsilon, S_\varepsilon$ is obtained by setting $\alpha = 0$ in equations (23)–(27), yielding

$$\nabla'^2 w_\varepsilon = \partial w_\varepsilon/\partial\tau,$$

$$\lim_{R\to 0}\,(-R^2\partial w_\varepsilon/\partial R) = 1,$$

$$\left.\begin{aligned} &w_\varepsilon = S_\varepsilon\\ &[\partial w_\varepsilon/\partial R] = \gamma\partial S_\varepsilon/\partial\tau\end{aligned}\right\}\text{ on } R = 1,$$

$$\lim_{R\to\infty} w_\varepsilon = 0,$$

with the initial conditions $w_\varepsilon = S_\varepsilon = 0$. Since the above problem is independent of the angle θ, we assert that $w_\varepsilon = w_\varepsilon(R,\tau)$ only, and this part of the problem is identical to that considered for u_1, S_1 in Section 2. Similarly, the problem for w_α, S_α is obtained by setting $\varepsilon = 0$ and expanding to order α, yielding

$$\nabla'^2 w_\alpha = \partial w_\alpha/\partial\tau, \tag{30}$$

$$\lim_{R\to 0}\,(-R^2\partial w_\alpha/\partial R) = 0, \tag{31}$$

$$w_\alpha = S_\alpha + \cos\theta, \tag{32}$$

$$[\partial w_\alpha/\partial R] = \gamma(2\cos\theta + \partial S_\alpha/\partial\tau), \tag{33}$$

$$\lim_{R\to\infty} w_\alpha = o(e^{\alpha X}). \tag{34}$$

The initial condition is for the temperature distribution to be the steady-state solution satisfying (31). We are therefore making no change in the power (as far as the solution w_α is concerned) and so we expect w_α to keep its steady-state form. Thus, we set $\partial/\partial\tau \equiv 0$ in (30) and (33). The remaining steady-state problem has been solved by Malmuth [4], who noted that the perturbation scheme breaks down for $R = O(1/\alpha)$, so that a singular perturbation treatment is required. We do not repeat the details of his calculation except for the results

$$S_\alpha = -1-(1+2\gamma/3)\cos\theta, \tag{35}$$

$$w_\alpha = \begin{cases} -1-(2\gamma/3)R\cos\theta, & R < 1, \quad (36) \\ -1-(2\gamma/3R^2)\cos\theta, & R > 1, \quad (37) \end{cases}$$

which holds except for $R = O(1/\alpha)$.

The complete solution of the first order problem is obtained by the superposition of the zero order solution $S_0 = 1$, together with the transient correction for a stationary source, $\varepsilon w_\varepsilon$, and the steady-state correction for a moving source, αw_α.

4. RESPONSE FOR A MOVING SOURCE WITHOUT LATENT HEAT

Under practical welding conditions the Peclet number, $\alpha = va/2D$, may be of order unity and it is also of interest to see how the response time varies around the pool boundary in this case. As remarked earlier, an analytic solution with latent heat included appears to be out of the question. However, we may determine what happens (at least) qualitatively by neglecting latent heat and solving the simpler problem of (23) subject to the condition

$$\lim_{R\to 0}(-R^2\partial w/\partial R) = 1+Q_1(\tau),$$

where $Q_1(\tau) = 0$ for $\tau < 0$, and condition (27).

We set

$$w(R,\theta,\tau) = w_0(R,\theta)+w_1(R,\theta,\tau), \tag{38}$$

and we note that it is not necessary in this case to restrict the magnitude of the fractional change in power. The steady-state temperature profile which satisfies (23) with $\partial/\partial\tau = 0$ is the well-known solution [5]

$$w_0 = e^{-\alpha R}/R. \tag{39}$$

Taking Laplace transforms, the transform of the solution is

$$\bar{w}_1(R,\theta;p) = (\bar{Q}_1/R)e^{-(\alpha^2+p)^{1/2}R}.$$

For a step change in power (Q_1 = const. for $\tau > 0$), this transform can be inverted to give

$$w_1 = (Q_1/2R)\{e^{\alpha R}\operatorname{erfc}(\tfrac{1}{2}R\tau^{-1/2}+\alpha\tau^{1/2}) + e^{-\alpha R}\operatorname{erfc}(\tfrac{1}{2}R\tau^{-1/2}-\alpha\tau^{1/2})\}.$$

On the molten boundary we have

$$w_0+w_1 = e^{\alpha R\cos\theta},$$

and the position of this boundary, $R = S(\theta,\tau)$, is given by

$$S = e^{-\alpha S(1+\cos\theta)} \times \{1+\tfrac{1}{2}Q_1[e^{2\alpha S}\operatorname{erfc}(\tfrac{1}{2}S\tau^{-1/2}+\alpha\tau^{1/2}) + \operatorname{erfc}(\tfrac{1}{2}S\tau^{-1/2}-\alpha\tau^{1/2})]\}, \tag{40}$$

which may be solved numerically by Newton's rule.

Figure 3 shows the variation of the boundary position with time for various angles for a 50% jump in power ($Q_1 = 0.5$) and $\alpha = 1$, e.g. a power of around 1 kW and a traverse speed of around 2 mm s^{-1}, in steel. The amplitude of the response is noticeably greater in the rear of the pool than in the front though equilibrium is approached more rapidly in the front. Figures 4 and 5, respectively, show how the time for the boundary to move halfway to its new equilibrium position, $\tau_{0.5}$ varies with the fractional change in power, Q_1, and the Peclet number, α, for various angular positions around the pool. From Fig. 4 we see that the characteristic response time is fairly sensitive to the size of the jump in power in the rear of the pool though not in the front. For example, for a change in power from 1 to 1.5 kW and a speed of 2.3 mm s^{-1} in steel, the (dimensional) response time varies from 0.4 to 1.8 s from the front to the rear of the pool.

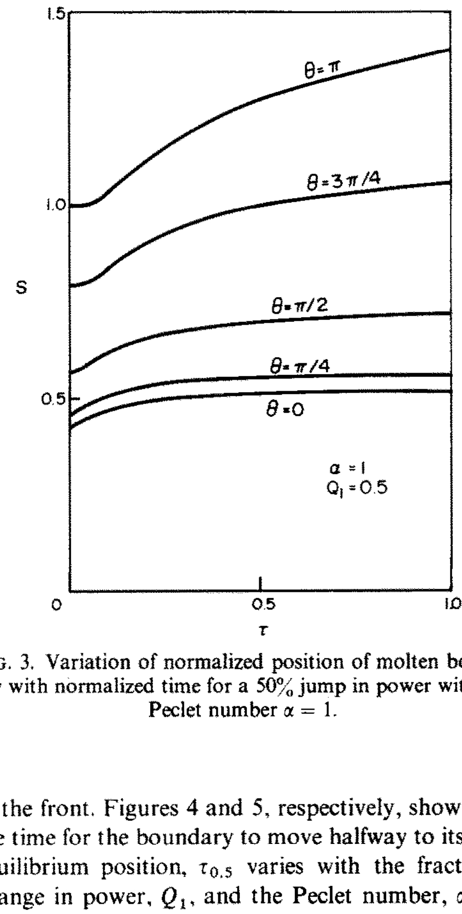

FIG. 3. Variation of normalized position of molten boundary with normalized time for a 50% jump in power with the Peclet number $\alpha = 1$.

5. DISCUSSION

It is important for automatic welding control to know how quickly the pool recovers its shape when the power is restored to its original value after some time. Let

$$Q_1(\tau) = \begin{cases} 0, & \tau \leqslant 0, \\ Q_1, \text{ const.}, & 0 \leqslant \tau \leqslant \tau_0, \\ 0, & \tau > \tau_0. \end{cases}$$

The equation for the position of the molten boundary, equation (40), becomes

$$S = e^{-\alpha S(1+\cos\theta)}\{1+Q_1[f(S,\tau)-f(S,\tau-\tau_0)]\},$$

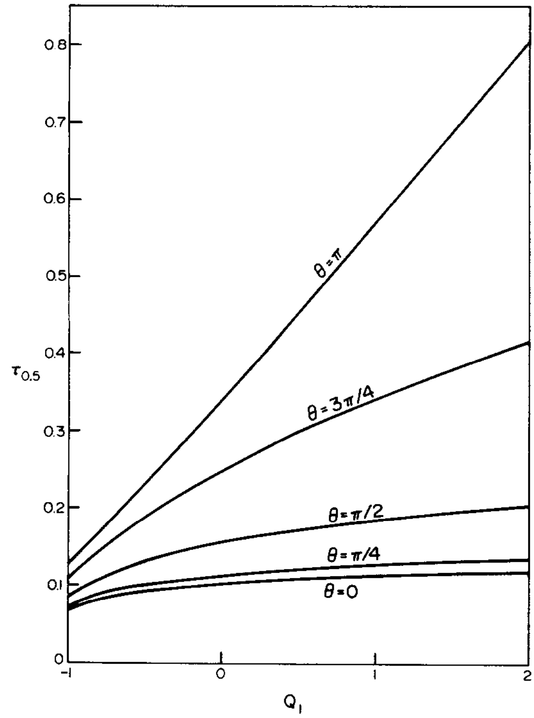

FIG. 4. Variation of normalized time for molten boundary to move halfway to new equilibrium position, $\tau_{0.5}$, against fractional jump in power for a Peclet number $\alpha = 1$.

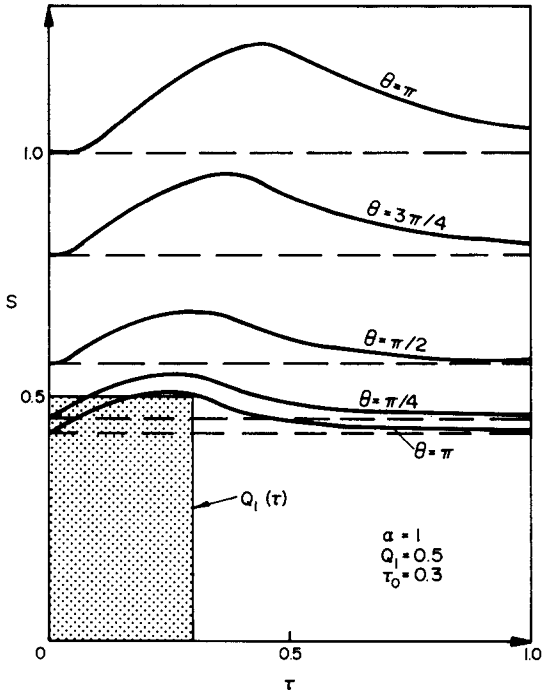

FIG. 6. Variation of normalized response of weld pool boundary to a square-wave disturbance in power.

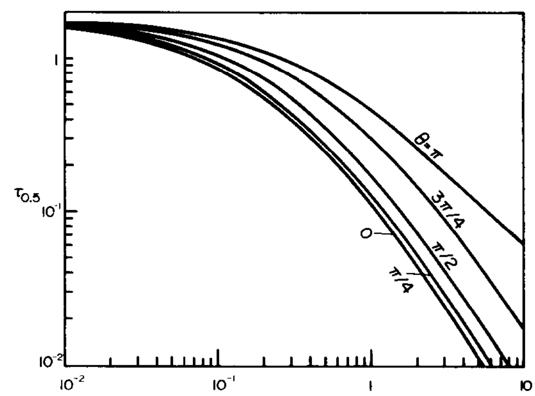

FIG. 5. Variation of normalized time for molten boundary to move halfway to new equilibrium position, $\tau_{0.5}$, against normalized Peclet number α, for a 50% jump in power.

where

$$f(S,\tau) \equiv e^{2\alpha S}\operatorname{erfc}(\tfrac{1}{2}S\tau^{-1/2}+\alpha\tau^{1/2}) + \operatorname{erfc}(\tfrac{1}{2}S\tau^{-1/2}-\alpha\tau^{1/2}).$$

Figure 6 shows the response of the pool boundary with $\tau_0 = 0.3$, $\alpha = 1$ and $Q_1 = 0.5$, for various angular positions around the pool. Again, the variation with angle is significant; in the front part recovery occurs around $\tau = 0.6$ whereas in the rear of the pool does not even start to move appreciably until $\tau \simeq 0.1$ and only recovers at about $\tau = 1$.

The implications for attempts to control welding processes automatically are significant. Instantaneous measurements of the length, width or area of the pool are not sufficient to specify the state of the system. Some recent history of the pool dimensions is also needed. In particular, data would need to be stored for about twice as long for a system based on length measurement than one based on the width, but the accuracy could be greater by a factor of three.

Acknowledgement—This paper is published with the permission of the Central Electricity Generating Board.

REFERENCES

1. N. Christensen, V. de L. Davies and K. Gjermundsen, Distribution of temperatures in arc welding, *Br. Weld. J.* **12**, 54–75 (1965).
2. N. D. Malmuth, W. F. Hall, B. I. Davis and C. D. Rosen, Transient thermal phenomena and weld geometry in GTAW, *Welding J.* **53**, 388S–400S (1974).
3. D. Longworth, A numerical method to determine the temperature distribution around a moving weld pool, in *Moving Boundary Problems in Heat Flow and Diffusion*, edited by J. R. Ockendon and W. R. Hodgkins, pp. 54–61. Clarendon Press, Oxford (1975).
4. N. D. Malmuth, Temperature field of a moving point source with a change of state, *Int. J. Heat Mass Transfer* **19**, 349–354 (1976).
5. D. Rosenthal, The theory of moving sources of heat and its application to metal treatments, *Trans. Am. Soc. Mech. Engrs* **68**, 849–866 (1946).

LA REPONSE DE LA ZONE FONDUE D'UNE SOUDURE AUX PERTURBATIONS DE PUISSANCE

Résumé—La commande automatique des procédés de soudage se développe rapidement et il est important de comprendre comment la frontière de la zone fondue répond aux changements de puissance. On montre comment le temps de réponse dépend de la chaleur latente et de la vitesse de déplacement de la source de chaleur. Pour une source se déplaçant lentement, la solution est la superposition de la solution transitoire pour une source stationnaire et de la solution permanente pour une source mobile. Lorsque le nombre de Péclet, $\alpha = va/2D$ (v = vitesse de déplacement, a = rayon d'équilibre d'une zone fondue stationnaire, D = diffusivité thermique) est $O(1)$, l'amplitude et le temps de réponse varient considérablement de l'avant à l'arrière de la zone fondue.

DIE REAKTION EINES SCHWEISSBADES AUF STÖRUNGEN IN DER ENERGIEZUFUHR

Zusammenfassung—Die automatische Steuerung von Schweißprozessen ist in schneller Entwicklung begriffen; hierfür ist es wichtig zu verstehen, wie die Grenze des Schweißbades auf Änderungen in der Energiezufuhr reagiert. In diesem Aufsatz wird über die Abhängigkeit der Reaktionszeit von der Schmelzwärme und von der Bewegungsgeschwindigkeit der Wärmequelle berichtet. Es wird gezeigt, daß bei einer langsam bewegten Wärmequelle die Lösung eine Superposition der instationären Lösung für eine ortsfeste Quelle und der stationären Lösung für eine bewegte Quelle ist. Wenn die Peclet-Zahl, $\alpha = va/2D$ (v = Bewegungsgeschwindigkeit, a = Gleichgewichts-Radius eines ortsfesten Schweißbades, D = Temperaturleitfähigkeit) die Größenordnung 1 hat, variieren die Amplitude und die Reaktionszeit von der Front der Schweißzone bis zu ihrem hinteren Rand beträchtlich.

РЕАКЦИЯ СВАРОЧНОГО ПРОПЛАВА НА ИЗМЕНЕНИЕ ПОДВОДА ЭНЕРГИИ

Аннотация—В настоящее время интенсивно развиваются методы автоматического контроля за процессами сварки, в связи с чем представляется важным выяснение влияния изменения подводимой энергии на поведение границы расплава. Исследуется зависимость постоянной времени от скрытой теплоты плавления и скорости перемещения источника тепла. Показано, что при наличии медленно перемещающегося источника решение задачи можно представить в виде суперпозиции переходного решения для неподвижного источника и стационарного решения для перемещающегося источника. При значении числа Пекле, $\alpha = va/2D$ (где v— скорость перемещения источника, a — радиус расплава в стационарном состоянии, D — температуропроводность) порядка единицы, амплитуда и постоянная времени существенно изменяются по длине шва.